## Compound Interest

Problem: You have accumulated some money from gifts, babysitting, and your paper route. You'd like to invest $\$ 500$ of it for four years so that you have some spending money available for college. There are three different banks in your area that all pay a $4.0 \%$ interest rate, but they each have a different compounding policy. At American Bank the interest is compounded annually, at Belmont National it is compounded quarterly, and at Bank of Carlisle it is compounded monthly. What impact does compounding have? Where should you invest your money?
Approach: A Microsoft Excel spreadsheet can help you find the answer to this problem. 1) Start Microsoft Excel if it isn't already running or choose New ... from the File menu. A new, blank spreadsheet window titled "Book l" appears.
2) If necessary, resize the Microsoft Excel window and the nested "Book 1" window to allow room to draw.
3) The interest rate given is expressed as an annual percentage rate (APR). We need to convert the APR to the interest rate per compounding period. To do this we must divide the APR by the number of compounding periods in a year.
4) Let's begin by creating a small table in the Excel spreadsheet. First we will create appropriate row and column headings.

b) In cells C1 and D1 enter Bank B and Bank C, respectively.
c) In cell A2, enter APR.
d) In cell A3, enter Periods/Yr.
e) In cell A4, enter Rate/Period.
5) Depending on the default settings in Excel, the last entry or two may have overflowed the width of the column A. We can fix this as follows:
a) Click on the "A" heading at the top of column A so that the entire column is highlighted, as shown at right.
b) Then with column A still selected, choose

Column_AutoFit Selection from the Format menu, as shown at right.

6) Now let's fill out this table with the information given in the problem.
a) In this case, the APR is the same for each bank, so enter $\mathbf{4 \%}$ in cells B2, C2 and D2. (While displayed as 4\%, Excel correctly treats these entries as 0.04.)
b) Now we must enter the number of times that interest is paid each year. Enter 1 in cell B3 for Bank A, $\mathbf{4}$ in cell C3 and $\mathbf{1 2}$ in cell D3. (Note that when you are entering data in consecutive cells in a row, you can use the tab key to advance between columns.)

c) We will use a formula to compute the Rate/Period. In cell B4, enter $=\mathbf{b 2} / \mathbf{b 3}$ as shown at right (before pressing enter).
d) The formula we have just entered is called a relative formula, which means that whatever cell we copy it to, it will divide the value from two cells above by the value from one cell above. Therefore, we can copy it to cells C4 and D4.
e) Holding the cursor over cell B4, click the right mouse button. А рорир тепи appears as shown at right.
f) Move the cursor to the Copy selection on the menu, as shown at right.
g) Then use the left mouse button to select the Copy choice. A "marching black ants" highlight appears around cell B4.
h) The formula from cell B4 is now "copied." To paste the formula into cells C 4
 and D4, simply highlight both those cells by depressing (but not releasing) the left mouse button in cell C4, dragging the mouse to cell D 4 , and then releasing the mouse button. This operation is known as "click and drag."
i) With cells C4 and D4 highlighted as shown at right, press the Enter key to complete the paste.

7) We should reformat the cells in row 4 as percentages to make them more readable.
a) Click and drag to highlight cells B4 to D4.
b) Right mouse click inside the highlight box and then choose Format Cells ... from the pop-up menu.
c) In the dialog box which appears, click on the Number tab (if it is not on top).
d) Then select Percentage from the Category list. (see right)
e) The default of 2 decimal places is fine.
f) Press the OK button on the dialog box to complete the formatting.


Percentage formats multiply the cell value by 100 and displays the result with a percent symbol.
8) Now we'll create a second table directly below the first (and coupled to the first) to compute how much money we will have accumulated after each interest period for each of the three banks.
a) Enter column headings in row 6 and initial values in row 7 as shown at right.
b) We can reformat the headings to align them better with the data below.
c) Click and drag to select cells A6 through D6.
d) As we did in 7(b), right mouse click, and select Format Cells ... from the pop-up menu.
e) This time choose the Alignment tab in the Format Cells dialog box.
f) In the Horizontal Text Alignment area of the dialog box, click on the down arrow (_) to reveal the dropdown list.
g) Select Right (Indent) for the alignment.
h) Click on OK to dismiss the dialog.

9) Now we'll enter some formulas in row 8 and use Excel's copy feature to complete the second table.
a) The greatest number of periods that we'll need to track any of our investments is 4 yrs 12 periods $/ \mathrm{yr}=48$ periods for Bank C.
b) In cell A8, enter $=\mathbf{A} 7+1$.
c) Right mouse click in cell A8 and select Copy from the pop-up menu.
d) Use "click and drag" to select cells A9 through A55. The spreadsheet will automatically scroll down as you near the bottom of the visible window.
e) Hit Enter to complete the paste operation.
f) In cell B8, enter $=\mathbf{B 7}+\mathbf{B} 7 * \mathbf{B} \mathbf{4}$. This formula is relative with respect to column, but because of the ' $\$$ ' before the 4 , it is partially absolute with respect to row. Regardless of where it is copied, it will add the value from one row above to the product of the value from one row above and the value in row 4 above.
g) Using steps c through e above as a guideline, copy the formula from cell B8 to cells B9 through B11.
h) Also copy the formula from B8 to cells C8 and D8.
i) One cell at a time, click on each of the five cells to which the formula from B8 was copied, and examine the cell function $(f x)$ value as
 displayed in the Excel frame (An example for cell B11 is shown at right).
j) What pattern do you see in the cell function value as you move down column B?
k) What pattern do you see in the cell function value as you move across row 8 ?
) For Bank B, we'll need 4 yr. 4 periods/yr $=16$ periods. So copy the formula from cell C8 to cells C9 through C23.
m) For Bank C, we need 48 periods, so copy the formula from cell D8 to cells D9 through D55.
n) The values in the three Amount columns need to be reformatted as currency.
o) Click and drag to select a block of cells from B7 in the upper left corner to D55 in the lower right corner.
p) Right click within the highlighted block to pull up the Format Cells dialog box.
q) This time choose the Number tab in the Format Cells dialog box, then select Currency from the Category list.
r) The top portion of the final spreadsheet should appear as shown at

| צ Microsoft Excel |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Eile View Insert Format Iools Data Window Help |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| E31 - fx |  |  |  |  |  |  |
| 질 Book1 |  |  |  |  |  |  |
|  | A | B | C | D | E | F |
| 1 |  | Bank A | Bank B | Bank C |  |  |
| 2 | APR | 4\% | 4\% | 4\% |  |  |
| 3 | Periods $/ \mathrm{Yr}$ | 1 | 4 | 12 |  |  |
| 4 | Rate/Period | 4.00\% | 1.00\% | 0.33\% |  |  |
| 5 |  |  |  |  |  |  |
| 6 | Period | Amount | Amount | Amount |  |  |
| 7 | 0 | \$500.00 | \$500.00 | \$500.00 |  |  |
| 8 | 1 | \$520.00 | \$505.00 | \$501.67 |  |  |
| 9 | 2 | \$540.80 | \$510.05 | \$503.34 |  |  |
| 10 | 3 | \$562.43 | \$515.15 | \$505.02 |  |  |
| 11 | 4 | \$584.93 | \$520.30 | \$506.70 |  |  |
| 12 | 5 |  | \$525.51 | \$508.39 |  |  |
| 13 | 6 |  | \$530.76 | \$510.08 |  |  |
| 14 | 7 |  | \$536.07 | \$511.78 |  |  |
| 15 | 8 |  | $\$ 541.43$ | \$513.49 |  |  |
| 16 | 9 |  | \$546.84 | \$515.20 |  |  |
| 17 | 10 |  | \$552.31 | \$516.92 |  |  |
| 18 | 11 |  | $\$ 557.83$ | \$518.64 |  |  |
| 19 | 12 |  | $\$ 563.41$ | $\$ 520.37$ |  |  |
| 20 | 13 |  | \$569.05 | \$522.11 |  |  |
| 21 | 14 |  | \$574.74 | \$523.85 |  |  |
| 22 | 15 |  | \$580.48 | \$525.59 |  |  |
| 23 | 16 |  | \$586.29 | \$527.34 |  |  |
| 24 | 17 |  |  | \$529.10 |  |  |
| 25 | 18 |  |  | $\$ 530.87$ |  |  |
| 26 | 19 |  |  | \$532.63 |  |  |
| 27 | 20 |  |  | \$534.41 |  |  |
| 28 | 21 |  |  | \$536.19 |  |  |
| 29 | 22 |  |  | \$537.98 |  |  | right.

10) The final value in each Amount column represents the value of your investment at the end of four years at the particular bank for that column.
11) What is your final balance in Bank $C$ after 4 years ( 48 months)? $\qquad$
12) Which bank will maximize the return on your investment? $\qquad$
13) How does the frequency of compounding affect the growth of an investment?
14) Which has a greater impact on the rate of growth of your investment, the change from annual to quarterly compounding, or the change from quarterly to monthly compounding?
15) To appreciate the power of your spreadsheet, try changing the annual interest rates (APR) for banks A and B so that all three banks produce the same final balance.
16) Then change the APRs back to $4.0 \%$ and try extending the spreadsheet to find the balances for all three banks after 6 years.

What are the Amounts:
Bank A: $\qquad$
Bank B: $\qquad$
Bank C: $\qquad$

